HSC STYLE HOMEWORK

NAME: .....

SEMINAR DAY & TIME: .....

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HSC Mathematics Extension 1

POLYNOMIALS I

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Draw diagrams in pencil
- Board-approved calculators may be used
- A table of standard integrals may be used
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value

Total marks – 84

Attempt Questions 1–7
All questions are of equal value

Marks

Question 1 (12 marks)

(a) Determine whether the following functions are polynomials or not, giving reasons for your answer

(i) $4x^4 - x^3 + 3x^2 - 1$ **1**

(ii) $x^{-2} + x$ **1**

(iii) $(x + 3)^3$ **1**

(iv) π **1**

(v) $4x - \sqrt{x}$ **1**

(b) (i) Verify by expansion that $x(x+1)(x-3)$ is a polynomial **2**

(ii) Hence or otherwise, find the roots of the equation $x^3 = 2x^2 + 3x$ **2**

Degree of Sum and Difference

Given $P(x)$ and $Q(x)$ with degree *m* and *n* respectively then:

- If $n \neq m$, then $\deg\big(P(x) \pm Q(x)\big) = \max(m, n)$
- If $n = m$, then $\deg\big(P(x) \pm Q(x)\big) \leq n$

Degree of Product

Given non-zero polynomials $P(x)$ and $Q(x)$ with degree *m* and *n* respectively, then:

$$\deg\big(P(x) \cdot Q(x)\big) = m + n$$

(c) Let $P(x) = x^2 - 3x + 1$, $Q(x) = x^3 + 5$ and $R(x) = x^3 - 3x$. State the degree of the polynomial

(i) $P(x) + Q(x)$ **1**

(ii) $R(x) - Q(x)$ **1**

(iii) $P(x)Q(x)R(x)$ **1**

End of Question 1

Marks

Question 2 (12 marks)

(a) If $P(x) = 7x + 3$ and $Q(x) = 2x^2 - x + 1$, find an expression for

(i) $P(x) + Q(x)$ **1**

(ii) $Q(x) - P(x)$ **1**

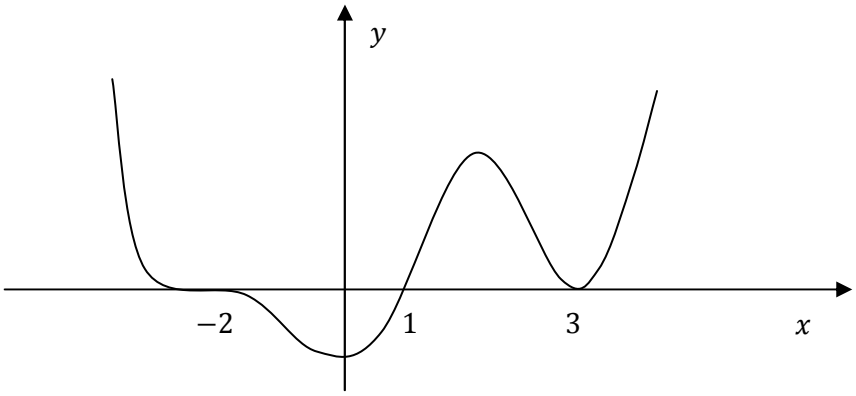
(iii) $P(x)Q(x)$ **2**

For large values of $|x|$, the leading term $a_n x^n$ dominates the function

Behaviour of Multiple Zeroes

- At a single zero, the curve cuts the *x*-axis, not tangent to it.
- At a zero of even multiplicity, the curve lies tangent to the *x*-axis without crossing it
- At a zero of odd multiplicity (≥ 3), the curve lies tangent to the *x*-axis and possesses a point of inflection at this root, crossing over the *x*-axis.

(b) The diagram below shoes the graph of $y = P(x)$



(i) State whether the degree of $P(x)$ is even or odd **1**

(ii) State whether the sign of the leading coefficient is positive or negative **1**

(iii) Hence, given that $P(x)$ is monic, find an expression for $P(x)$, giving your answer in factorised form **2**

(c) Without the aid of calculus, sketch the graphs of the following polynomials, clearly indicating any intercepts with the x-axis

(i) $y = (1 - x)(1 + x)(3 + x)$ **2**

(ii) $y = x^2(3 - x)$ **2**

End of Question 2

Marks

Question 3 (12 marks)

(a) (i) Without using calculus, sketch the graph of $y = x^3(x-1)^2$ **2**

(ii) Hence, find the values of x for which $x^3(x-1)^2 > 0$ **2**

(b) Using long division, divide the following, giving your answer in the form –

$$P(x) = D(x)Q(x) + R(x)$$

(i) $(2x^2 - 4x + 1) \div (2x + 1)$ **2**

(ii) $(3x^4 - x^3 - 6x - 1) \div (x - 1)$ **3**

(iii) $(x^5 - 3x^4 + x^3 + 4x - 1) \div (x^2 + 1)$ **3**

End of Question 3

Marks

Question 4 (12 marks)

(a) Given that $P(x) = x^3 - x + 2$, $Q(x) = x^4 - 3x^3 - x - 3$ and $R(x) = x + 1$, find

(i) $P(x) + Q(x) + R(x)$ **2**

(ii) $P(x)R(x)$ **2**

(iii) The degree of $P(x)Q(x)R(x)$ **1**

(iv) $Q(x) \div R(x)$ **3**

(b) Consider the polynomial $P(x) = x^3 - 3x - 2$

(i) Using long division, show that $P(x) = (x + 1)(x^2 - x - 2)$ **2**

(ii) Hence, without using calculus, sketch the graph of $y = P(x)$, showing any important information **2**

End of Question 4

Marks

Question 5 (12 marks)

(a) Use long division to express $4x^3 - 7x - 8$ in the form $Q(x)(x-1) + c$, where c is a constant. **3**

(b) Find real numbers A and B such that $x^2 - 2x + 7 \equiv A(x-1) + B(x^2+1) + C$ **2**

(c) (i) Without aid of calculus, sketch the graph of $y = 2x(x-3)^2(x+1)^3$ **2**

(ii) Hence or otherwise, solve the inequality $2x(x-3)^2(x+1)^3 < 0$ **2**

(d) Find $(9x^4 - 6x^2 + 3x - 2) \div (x^2 - 4)$ **3**

End of Question 5

Marks

Question 6 (12 marks)

(a) Neatly sketch the graph of $y = x^3(x-3)^2(x+1)$, showing all relevant information **2**

(b) (i) Expand and simplify $(x+4)(x+1)(x-3)$ **1**

(ii) Hence, solve $x^3 + 2x^2 < 11x + 12$ **2**

(c) Find the remainder when the polynomial $P(x) = x^6 - 4x^4 + 3x^3 + 3$ is divided by $x^3 + 1$ **3**

(d) Consider the function $F(x) = \frac{x^4-2x^2-3x-2}{x^2-x-2}$

(i) Prove that $F(x)$ is a polynomial **3**

(ii) Hence, find $\int F(x)dx$ **1**

End of Question 6

Marks

Question 7 (12 marks)

(a) Consider the graph of $y = x^4 + 3x^3 - 4x$

(i) Given that the graph passes through $(1, 0)$, find the coordinates of any other x-intercepts **4**

(ii) Sketch the graph, showing all relevant information **2**

(iii) Hence, solve the inequality $y > 0$ **2**

(b) Find $\int \frac{2x^5-4x^4-11x^3+29x^2-16x+7}{(x-2)^2}\,dx$ **4**

End of paper